

INPUT-OUTPUT MODEL AND ANNUAL SECTORAL WAGE RATES IN NIGERIA

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Abstract

An open input-output model was used with a linear programming formulation by (1963) to compute optimal sectoral wages for thirty (30) sectors of the Nigerian economy based on the 1990 base year input-output table. The Hadley (1963) linear programming formulation seeks to maximize labour requirements (z) which is given by Dw , subject to the constraints: $(1 - A)w \leq 9_0$ and $w \geq 0$. We obtained optimal wage rates by solving the linear programming formulation that comprised thirty (30) decision variables and thirty (30) slack variables. We used only 30 active sectors of the Nigerian economy out of 33 (indicated in the 1990 base year classification). The result of the study was that distributive trade sector (sector 24) had the highest annual wage rate of ₦129,580 whereas fabricated metal sector had the lowest optimal wage rate. The optimal annual wage rate for the other twenty eight (28) sectors lie within the range of ₦23,900 and ₦129,580. The sensitivity analysis report on the objective function coefficient for each sector showed the lowest minimum value of ₦44,192.49 million for the fishing sector (sector 3) and the highest minimum value of zero for producers of government services sector (sector 30). The minimum value of the objective function coefficient for the other twenty eight (28) sectors lie between ₦44,192.49 million and zero. Also, from the sensitivity report on the sectoral objective function coefficient, it was observed that the maximum objective function coefficient for all sectors were infinity (∞). This implies that there exist broad ranges within which the objective function value for each sector of the Nigerian economy can vary without altering the optimal solution of the linear programming problem. A sensitivity analysis report on the resource availability or right hand side (RHS) values of each sector's constraint inequality expression showed the lowest minimum RHS value of ₦0.02526 million for sector 26 (finance and insurance sector) and the highest minimum RHS value of zero for sectors 4 (forestry), 18 (vehicle assembly) and 30 (producers of government services). On the other hand, the lowest maximum RHS value of ₦1.28746 million was obtained for sector 1 (agriculture) whereas the highest maximum value of infinity (∞) was obtained for sectors 3 (fishing) and the twenty one (21) other sectors. This implies that these twenty two (22) sectors of the Nigerian economy, there are wide ranges within which the RHS values of the constraint inequality expressions can vary without altering the feasibility of the linear programming problem.

Among all the thirty (30) sectors of the Nigeria economy investigated, it was observed that crude petroleum sector (section 5) had the highest dual price of ₦87586.19 (which for our maximization problem is same as shadow price). The next highest dual (shadow) price was ₦79259.41 for agriculture sector (section 1) with the lowest dual price of ₦842.41 observed for forestry sector (section 4). We concluded from this result that crude petroleum and agriculture sectors contribute more to the economic well-being of the Nigerian economy than the forestry sector because a unit increase in the resource availability of crude petroleum and agriculture sectors result in more increase in the objective value (labour requirement) than a unit increase in the resource availability in the forestry sector. We also concluded that our contribution to knowledge by our analysis on the Nigerian economy is that the result of annual GDP compilations which had only a statutory or at best an archival value in the past, had by our work, achieved an increased status of being a tool for further economic analysis to provided an increased understanding of the Nigerian economy.

Key words: Open and closed input-output model, linear programming, sensitivity analysis value-added, final demand or bill of goods Leontief matrix, input-output table of Nigeria economy.

Introduction to the inception, theory and extension in the application of input-output model

Input –output model is most identified with the works of Wassily Leontief (1990-1999). Leontief initiated his work on input –output model in the 1930s for which he won a Nobel memorial prize in economic sciences in 1973 for using the open version of the model to explain the economy of the United States of America. One notable practical result of Leontief's pioneering input-output table is at it was used in making successful projections of post second world war employment growth in the year before 1950.

Leontief's (1951) pioneering efforts in the capacity of a consultant for the United States Bureau of Labour Statistics resulted in his 1947 publication of a 50 sector input output table that exposed more inter-industry relations in the United States economy. Later, Leontief (1951) developed a 500 sector input-output table and used it to conduct a comprehensive study on how the 500 sectors of the United States economy then inter-related with each other.

Extended use of input –output model

Besides the initial application of input-output model by Leontief (1951) for explaining national economies, today input-output model is used in the compilation of national accounts environmental studies and technological change forecasts. Other extensions of input-output analysis today include; estimate of the inflationary consequence of wage settlements, the direct and indirect impact of armament expenditures, estimate of capital requirements for economic development.

More recently, input-output model had been also applied to issues of world-wide economic growth and its environmental consequences; its impact on the world reserve of natural resources and the political and economic relations between the economies of developed and less-developed countries. Still other extensions of input-put analysis, include inter-regional input-output analysis (IRIO), multi-regional input-output analysis (MRIO) that extends across 15 regions consisting of 45 sectors each with balance of trading accounts; industrial ecology to develop new approaches to reducing, reusing and recycling of generated wastes while simultaneously conserving energy and converting materials of mineral and biological source into useful products (Duchin, 1992); the optimization frame-work for identifying the least cost technological options faced by different sectors of the economy (Duchin, 1992); in formulating the “World economy input-output model” which includes international flows of goods and services, financial capital, and people (Duchin, 1992); and in the analysis of environmental implications of consumption (even for trade between different countries) mainly to assess green house emissions, land use, and water use with a view to quantifying the total land, carbon and water foot-prints of a product.

Theoretical framework and the mathematics of input-output Analysis

For a three-industry model in which the economy is assumed to consist of three industries; L_1 , L_2 and L_3 . Suppose that each of the three industries produces different types of products (which are homogenous for each industry) so that no product is produced jointly by two sectors nor can one industry produce two types of homogenous products. Furthermore, we assume that a part of industry L_1 's production is used by each of the three industries, while the remainder is used up by non-production-related consumers as “final demand”.

The same is true of production of industry L_2 and industry L_3 . The hypothetical table below shows how these three industries inter-relate.

Table 1: Input-out matrix for a hypothetical three sector economy showing the flow of goods and services. All values are in millions of Naira. Adapted from: Agbadudu, A. B. (1998:235). Mathematical methods in Business and economics 2nd ed. Lagos: Lagos University Press;

Input purchases by	L_1	L_2	L_3	Final demand	Total
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Output purchases by					
L ₁	50	20	40	70	180
L ₂	20	30	20	90	160
L ₃	30	20	20	50	120
Value –added	80	90	40		
Total	180	160	120		

Reading across the rows of the table shows how each industry disposes of its output. For example considering the first row, out of the ~~A~~180m worth of goods produced by industry L₁, ~~A~~50m is used by industry L₁, ~~A~~20m is used by industry L₂ and ~~A~~40m is used by industry L₃ for the production of goods and services in each of the industries while ~~A~~70m is purchased by consumers (as final demand) for their non-production -related consumption. The second and third rows can be interpreted in self-same way. Reading down the columns of the table, shows that for industry L₁'s process, ~~A~~50m worth of inputs used, was generated by industry L₁ and industry L₁ bought ~~A~~20m and ~~A~~30m worth of goods and services from industries L₂ and L₃ respectively.

Table 1 can be converted into table of technical coefficients (α_{ij}) by dividing the entries in each column by its industries total to get:-

$$\begin{array}{l}
 L_1 \\
 L_2 \\
 L_3
 \end{array}
 \begin{array}{|c|c|c|}
 \hline
 L_1 & L_2 & L_3 \\
 \hline
 50/180 & 20/160 & 40/120 \\
 20/180 & 30/160 & 20/120 \\
 30/180 & 20/160 & 20/120 \\
 \hline
 \end{array}
 =
 \begin{array}{l}
 L_1 \\
 L_2 \\
 L_3
 \end{array}
 \begin{array}{|c|c|c|}
 \hline
 a_{11} & a_{12} & a_{13} \\
 a_{21} & a_{22} & a_{23} \\
 a_{31} & a_{32} & a_{33} \\
 \hline
 \end{array}$$

For an 'n' industry model, the table of technical coefficient can be shown as;

$$\begin{array}{l}
 L_1 \\
 L_2 \\
 \cdot \\
 L_3 \\
 \cdot \\
 \cdot \\
 L_n
 \end{array}
 \begin{array}{|c|c|c|c|c|}
 \hline
 L_1 & L_2 & L_3 & \dots & L_n \\
 \hline
 a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\
 a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\
 \cdot & \cdot & \cdot & \cdot & \cdot \\
 a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\
 \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot \\
 a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \\
 \hline
 \end{array}$$

For the n-industry case, suppose y_1 represents the total of output from all industries required for production in industry L1, y_2 represents the total output from all industries required for production in industry L2 and y_n represents the total output from all industries required for production industry L_n. Suppose also, that industry L1 produces a total output of X_1 units, industry L2 produces a total output of X_2 and industry L_n produces a total output of X_n . Note that total output (X_i) comprises both production –related consumption (Y_i) and non-

production –related consumption (that is final demand or bill of goods). d_i ; Then the following input-output equations would apply if we do not include final demand in our consideration.

$$\begin{aligned}
 Y_1 &= a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \dots + a_{1n}X_n \\
 Y_2 &= a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \dots + a_{2n}X_n \\
 Y_3 &= a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + \dots + a_{3n}X_n \\
 Y_n &= a_{n1}X_1 + a_{n2}X_2 + a_{n3}X_3 + \dots + a_{nn}X_n
 \end{aligned}$$

Putting this in matrix form gives:-

$$Y = AX \quad \text{----- (1)}$$

Where:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \cdot \\ \cdot \\ Y_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \cdot \\ \cdot \\ X_n \end{bmatrix}$$

If we treat each industry as a sector of an economy and also if all the goods produced by a sector are used by only goods-producing sectors with no consumer sector to consume some of it, the input-output model is called a closed model. The closed input-output model is shown in equation 1. However, if a consumer sector exists for each sector so that each industry produces for its use and the use of other sectors together with a portion consumed by the consumer sector, the input-output model is called an open model. Equation 2 shows an open input-output model. Assuming that the demand vector showing the final demand sector for each sector is D where:

$$D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \cdot \\ \cdot \\ d_n \end{bmatrix}$$

$$D = X - AX = X(1 - A) \quad \text{Therefore } X = (1 - A)^{-1} D \quad \text{----- (2)}$$

Where;

- D = final demand vector showing the final demand value for each sector
- A = technical coefficient vector showing technical coefficient a_{ij} which shows the monetary value of goods and services produced in sector and consumed for production–related activity in sector j (expressed as a fraction of the total output of sector j).
- X = Vector representing the total amount of production for each sector
- I = an identity matrix
- (1 - A) = Leontief matrix
- (1 - A)⁻¹ = matrix of inter-dependence coefficients or inverse of Leontief matrix

α_{n1} = units of the output of sector “n” required by sector 1 to produce one unit of product in sector 1

Y = Vector showing the total output from each sector which other sectors (including the sector supplying the output) require for production (excluding the final demand)

Literature Review and theoretical framework

Out of the various but non-exhaustive applications (i.e extensions) of input output model, we concern ourselves with the use of input-output model to compute optimal sectoral wages in thirty (30) sectors of the Nigerian economy. The sectoral wage rates are called optimal because they result (as solution) from a linear programming technique which uses 30 decision variables (w_1 to w_{30}) and 30 slack variables (S_1 to S_{30}).

If we assume that only a single wage (w) exists in the economy and that a_{oj} units of labour are required to produce one unit of product j, then the labour cost for one unit of product is $a_{oj}w$ and hence v_j can be expressed as:-

$$V_j = w_{a_{oj}} + S_j \text{----- (3)}$$

Where:

S_j = Value added from other primary factors besides wages (like profits, interest, rent, dividends, depreciation, indirect taxes and purchases from foreign suppliers or imports).

$W_{a_{oj}}$ = Value added from only wage payments

We would make a further assumption that in the economy under consideration, labour is the only primary contribution to value added (that is there is no profits, no interest and no rents payments etc).

McConnell (1963:538) corroborates this assumption that labour is the primary contribution to value added when he explained that when the word “labour is broadly defined to include all payments received by all categories of workers, wages would clearly amount to more than two-thirds of the National income”.

By the enabling assumption that only labour contributes to value added, equation 3 simplifies to:

$$V_j = W_{a_{oj}} \text{----- (4)}$$

Price vector from input –output mathematics can be shown as:-

$$P = (1 - A^1)^{-1} V \text{----- (5)}$$

It is pertinent to point out that by a basic theorem of matrix algebra, inverse of the transpose of a matrix is equal to the transpose of the inverse of that same matrix. This means that:

$$[(1 - A^1)]^{-1} = [(1-A)^{-1}]^1 \text{----- (6)}$$

Therefore equation 5 can be re-expressed as:-

$$P = [(1-A)^{-1}]^1 V \text{----- (7)}$$

Combining equations 4 and 7, equation 7 can be written as:

$$P = W_{a_{oj}} [(1-A)^{-1}]^1 \text{----- (8)}$$

Hadley (1963) reported that the price represented by equation 7 can be thought of the prices which would obtain if the economy operated under pure competition and in a state of long run equilibrium. Hadley (1963) used the variables and requirements of input-output analysis to formulate a primal linear programming problem that seeks to minimize labour requirements (Z) subject to the production of at least the bill of goods (final demand) of D as:-

$$\text{Minimize } Z = \alpha_0 x$$

Subject to the constraints: $(1-A) X \geq D, X \geq 0$ ----- (9)

Where:

Z = labour requirement

$\alpha_0 = \alpha_{01}, \alpha_{02} \dots \alpha_{0n}$

(1-A) = Leontief matrix vector

X = total output vector

D = bill of goods or final demand vector

The dual of the primal linear programming problem stated above can be expressed as:-

Maximize $Z = D^1 W$

Subject to the constraints: $(1 - A)^1 W \leq \alpha_0', W \geq 0$ ----- (10)

If actual values of D, A and α_0 are plugged into the linear programming formulation represented in equation 10, thirty (30) optimal sectoral wages can be obtained (see table 1 in the appendix for a complete formulation of the problem).

Research Methodology

The philosophical basis of the study was to use input –output model together with linear algebra and linear programming technique to compute optimal wages of 30 sectors of the Nigerian economy. Input-output table that shows technical coefficients (a_{ij}), final demand (D), value –added (V) and total output (X) for the Nigerian economy based on the 1990 base year classification which was compiled by the National Bureau of statistics (NBS) was used for the study. The linear programming formulation of equation 10, gave the following structure:-

Maximize $Z = D_1 W_1 + D_2 W_2 + D_3 W_3 + \dots + D_{30} W_{30}$

Subject to the constraints:

$$(1 - \alpha_{11}) W_1 - \alpha_{12} W_2 - \alpha_{13} W_3 = \alpha_{14} W_4 \dots \alpha_{1,30} \leq \left(\frac{v_1}{x_1} \right) W$$

$$- \alpha_{21} W_1 + (1 - \alpha_{22}) W_2 - \alpha_{23} W_3 - \alpha_{24} W_4 \dots \alpha_{2,30} W_{30} \leq \left(\frac{v_2}{x_2} \right) W$$

$$\begin{matrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{matrix}$$

$$- \alpha_{30,1} W_1 + \alpha_{30,2} W_2 - \alpha_{30,3} W_3 - \alpha_{30,4} W_4 \dots + \alpha_{30,30} W_{30} \leq \left(\frac{v_{30}}{x_{30}} \right) W$$

Where:

W = average economy –wide wage rate estimated as the annual wage of salary grade level 07 in Nigeria (about ₦90,000 per year) for 1990.

X_1 = total output from sector 1

V_1 = value added for sector 1

D_1 = final demand for sector 1

W_1 = optimal wage for sector 1

A more detailed copy of the linear programming formulation containing the actual data is shown in appendix as table 1.

Findings and Implication

Primarily, the analysis of data comprised the computer solution to 60 variable linear programming problem. The findings of the study are displayed in the appendix as tables 2,3,4 and 5; summary of the findings that resulted from the analysis of data:-

1. Expectedly optimal sectoral wage rates per annum differed from sector to sector in the Nigerian economy. This corroborates the result of the empirical framework of the work of Mincer (1958, 1974) and Becker (1964) on human capital models of earnings determination. The “Mincerian” model of earning determination is based on the premise that observed wage differences among individuals are brought about by a combined effect of school and post-school investments (training and work expenses) and a host of other socio-economic factors like geographical location, marital status, and nationality which are expected to be correlated with earnings. Hence wage rates are expected to differ.
2. Distributive trade sector had the highest optimal wage rate of ₦0.12958 million (₦129,580) whereas fabricated metal sector had the lowest optimal wage rate of ₦0.0239 million (₦23,900)
3. The annual wage rates for the other twenty eight (28) sectors lie within the range of ₦23,900 and ₦129,500.
4. Among all the 30 sectors investigated, the highest dual price (or shadow price for this maximization problem) was ₦87581.19 for crude petroleum sector (sector 5) and the next to highest shadow price was ₦79259.41 for agriculture sector (sector1). On the other hand, the lowest dual (or shadow) price of ₦842.41 was obtained for forestry sector (sector 4). This implies that a unit positive change in the resource availability (RHS) of crude petroleum and agriculture sectors would be more beneficial to the economic well-being of the Nigerian economy than a unit positive change in the RHS of forestry, sector. In other words, the Nigerian economy is more dependent on the crude petroleum and agricultural sectors than the other 28 sectors (including forestry sector). Before ₦79259.41 price of ₦642.41.
5. The sensitivity analysis report on the objective function coefficient for each sector showed a minimum value as low as- ₦44,192.49 million for the fishing sector (sector 3) and a minimum value as high as zero for producers of government services sector (sector 30).
6. The minimum value of the objective function coefficient for other twenty eight (28) sectors lie between -₦44,192.49 million and zero.
7. The lowest maximum value of the RHS of the constraint inequality was ₦1.28746 million (for agriculture sector) whereas the highest maximum value of infinity (∞) was obtained for the following 21 sectors:- fishing, forestry, crude petroleum, other mining, food, foot-wear and leather, wood, refineries, rubber and construction, transport, communication, distributive trade real estate and business service, housing (dwellings, community social and personnel services and producers of government services sectors. This means that the 21 sectors (which constitute majority of the sectors) have wide ranges within which the RHS values of the constraint inequality can vary without distorting the feasibility of the linear programming problem. This implies that the optimal solution (wage rates) are very stable, rebuts and resilient

We expected crude petroleum sector (sector 5) which has the highest sectoral value-added figure of N86618.03 million to produce the highest optimal annual wage rate. Besides, workers in the crude petroleum sector which are locally referred to as “oil company workers” are noted for receiving very high wages in Nigeria.

In practice however, the crude petroleum sector has an optimal wage rate as low as: N0.05120 million (N51,200) whereas distributive trade sector with a value-added figure as low as N35837.66 had the highest optimal wage rate of N0.129588 million (N129,580).

Most recent studies with input –output framework in Nigeria are not of the type done in this work. The nearest work to ours in the Nigerian literature is that by Agbadudu, Ogunrin, Ighomereho (2004) in which optimal wages were determined in a very aggregated version of the Nigerian economy. Rather than using 30 sectors they aggregated the number of Nigerian sectors to only five (5) with concomitant problems that usually bedevil excessive aggregation in economic analysis results. We adopted Agbadudu, Ogunrin and Ighomereho's (2004) approach of assuming an average annual wage rate of N90,000 (or N0.09 million) for the Nigerian economy.

Based on some observations made from the Agbadudu, Ogunrin and Ighomereho's (2004) work, concerted effort on dimensional analysis was done to ensure that the dimension of the constraint inequality expression of the Right Hand Side (RHS) and that of the Left Hand Side (LHS) of each constraint inequality were consistent. We also ensured that N90,000 (average economy-wide wage rate per annum assumed for the study) was expressed in millions of Naira as other inputs to the linear programming problem like final demand, value-added, total demand etc.

For Agbadudu, Ogunrin, Ighomereho's (2004) study, no sensitivity analysis was done on their linear programming problem to get both shadow prices for each constraint inequality expression and the range within which the objective function coefficient can vary without altering the optimality of the solution. The fact that our study and that of Agbadudu, Ogunrin and Ighomereho (2004) do not use the same number of sectors, made it difficult to compare the results.

Agbadudu, Ogunrin and Ighomereho's (2004) wage rate results were, N15.34 million for services sector, N14.32 million for agriculture sector, N3.84 million for manufacturing sector and N4.26 million for agricultural processing sector. Though the results show disparity in sectoral wage rates, yet the sectoral values are too high for annual wage rates for individuals in any sector.

It is pertinent to re-iterate that it is normal to have wage differentials for workers in different sectors of an economy; for workers in the same sector of the economy, and between workers in public and private sectors of the same country. Gunderson (1979) justified the necessity for wage differentials between public and private sector workers by stating that wage rate management in the public sector is based more on political consideration whereas wage rate in the private sector is based more on profit consideration. In support of this rationalization for the inevitability of wage rate differentials between public and private sector workers, Cousineau and Lacroix (1977) argued that government is less mindful about wage rates even when the rate is outrageously high because of government's ability to raise money through taxes and borrowing.

We believe that the government is in a stronger position to pay higher salaries in Nigeria because of "oil money" but the activities of politicians who make the decisions are often not in the interest of the welfare of the generality of Nigerian workers. One evidence to substantiate this charge is that the salary of elected politicians is always outrageously higher than necessary when compared to general workers salary in Nigeria. The second evidence of this contention is

that before the 1998 wage review, public sector wage in Nigeria stagnated since 1993 inspite of rapidly increasing general price level and income to the Nigeria government (Aminu, 2008).

The optimal wage rate obtained from this work seems to be more closely related to both the size of the Labour force and hence the value of the value-added for each sector. For example, the value-added for crude petroleum and agriculture sectors are respectively ₦86618.03 million and ₦68,370.70 million. Also, Nigeria labour force by occupation is 32% for services, 30% for agriculture and 11% for manufacture (Aminu, 2008).

Conclusion

We conclude that our procedure which generates sectoral wage rates using data from the economy could be used in providing input into the process of proffering solution to wage rate determination problem in the economy. When both organized labour and employers of labour including government are convinced that the optimal sectoral wage rates result from parameter of the economy, they would more likely be prepared to accept with little or no hesitation and complaints. Finally, we also concluded that we contributed to the knowledge of GDP analysis of the Nigerian economy by converting the result of statutory annual GDP compilation into a tool for further economic analysis with the potential of providing increased understanding of the Nigerian economy.

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