PERMUTATION SYMMETRY GROUPS TECHNIQUE FOR COUNTING SWITCHING CIRCUITS

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Abstract

The symmetry positions of geometric objects are of useful applications in switching circuits. Permutation symmetry groups are used in counting the number of switching circuits. The permutations acting on geometric objects give description of their symmetries. By permutation of geometric objects, we mean the symmetry positions obtained by rotation (or reflections or a combination of both) through some specified angle so that the new positions of the objects make them indistinguishable from the original positions. These symmetry positions correspond to be distinct number of symmetries. For an n-sided object, every rotation corresponds to a permutation of n distinct objects. Since we are living in a three-dimensional world, these symmetry groups play a crucial role in the application of modern algebra to electricity (switching circuits).
Keywords: Permutation, Symmetry groups, switching circuits, geometric objects, counting.

Introduction

Symmetry groups are groups composed of all rigid motions or similarity transformation of some geometric objects onto themselves. These groups are of two types namely: the Finite or Discrete and Lie symmetry groups. The symmetry positions of geometric objects are of useful applications in electricity (counting the number of switching circuits).

Symmetry groups arise everywhere in nature: In crystallography in describing the external and internal spatial structure of crystals. Elementary particle physics used SU (3) as a symmetry group of elementary particles which led to the discovery of a fundamental particle called Omega-minus baryon. With the aid of permutation groups, one can find the number of different chemical compounds obtained by attaching a CH_3 or H radical to each carbon atom in the benzene ring. In molecular vibrations, more complicated molecules are invariant only under a (finite) crystallographic group, for example, the benzene ring (hexagonal symmetry) or ammonia molecules and the energy levels in atoms can be determined using symmetry groups in theory of electron structure. In quantum physics there is a built-in linear structure through which symmetries are automatically realized by group representations, flowers have five petals with symmetry group C_5, during cell division, a cell is splitted into two symmetrical cells, snowflakes (a small fat six-sided bit of frozen water that falls as snow) have a symmetry group of D_6, musical vibrations have symmetrical sound (echo), the work habit of bees in hive displayed a sort of symmetrical stages and Romanesque Cathedral (Ancient building in the Western Europe in about 11th century) has symmetrical structures.

From a historical point of view, in the early work on groups, before their abstract nature was fully realized, the only type to be studied was that of the permutation groups. That is, subgroups of the symmetry groups. Many of the properties of general finite groups were discovered for the permutation groups. Early workers in this connection being Cayley, Cauchy and Galois, all in the nineteenth century, in a certain sense all finite groups are contained in the symmetry groups. Hence, modern algebra plays a fundamental role to real life situation.

Theoretical Analysis

For any set n, the group of transformations (permutations) of n is a symmetric group on n and is denoted by S_n. If n is a finite set having n elements we call S_n the symmetric group of degree n. The symmetry group is usually the full permutation group S_n.

Theorem 1.1 Polya-Burnside Method of Enumeration

This method was first discovered by W. Burnside in 1911 and in 1937 G. Polya discovered the applicability to many combination problems.

This method detects the number of orbits of a set under the action of a symmetry group. Let G be a finite group that acts on a finite set n.

The Burnside's theorem describes the number of orbits in terms of the number of elements left fixed by each element of G.

Theorem 1.2: Burnside Theorem

Leg G be a finite group that acts on the elements of a finite set X. For each $g \epsilon G$, let fix g {x ϵ X: g(x) = x}, the set of elements of x left fixed by g. If N is the number of orbits of x under G, then,

$$N = \frac{1}{\#G\Sigma\#fix\ g}$$
$$gEG$$

Proof: We count the set S ={(x,g) ϵ X x G: g(x)=x} in two different ways. Consider Table 1.1 whose columns are indexed by the elements of X.

Table 1.1: The elements of S correspond to the ones in this table

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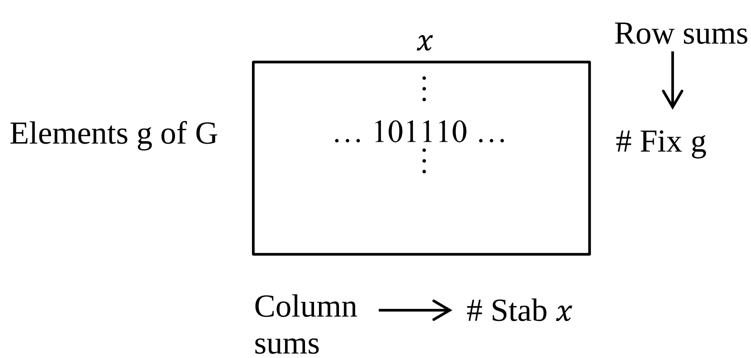

and whose rows are indexed by the elements of G. Put a value of 1 in the (x,g) position if g(x) =x; otherwise, let the entry by 0.

The sum of the entries in row g is the number of elements left fixed by g, # fix g. The sum of the entries in columns x is # stab x, the number of elements of G that fix x.
We can count the number of elements of S either by totaling the row sums or by totaling the column sums. Hence,

#S=Σ#fixg = Σ# stab x
gEG xEX

Choose a set of representations, $x_1, x_2 \dots x_N$, one from each orbit of X under G. If x is in the same orbit x_1, then orb x = orbx_i, then #stab x = #stab x_i.

Hence, $\sum_{\mathrm{gEG}}$#fixg = $\sum_{i=1, x \in \mathrm{orb}x_1}^{N} \Sigma \#\mathrm{stabx}$ = $\sum_{i=1}^{N}$ (# Orbx_i)(#stabx_i)
= N.(#G)

Methodology
Permutation Symmetry Groups Technique for counting switching circuits

Consider the different switching circuits obtained by using three switches. We can think of these as black boxes with three binary inputs x_1, x_2 and x_3, and one binary output $f(x_1, x_2, x_3)$ as in Figure 1.1. Two circuits, f and g are called equivalent if there is a permutation II of the variable so that $f(x_1, x_2, x_3) = gf(x_{\text{II}1}, x_{\text{II}2}, x_{\text{II}3})$. Equivalent circuits can be obtained from each other by just permuting the wires outside the black boxes, as in Figure 1.2.

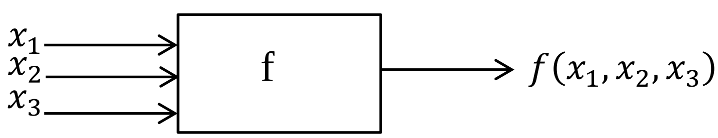

Fig. 1.1: A Switching Circuit

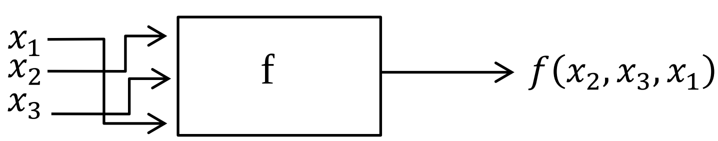

Fig. 1.2: A permutation of the Inputs

For example, find the number of switching circuits using three switches that are of equivalent under permutations of the inputs. There are eight possible inputs using 2^8 = 256 circuits to consider. The symmetric group S_3 acts on these 256 circuits, and we wish to find the number of different equivalence classes; that is, the number of orbits.

Table 2 lists the number of circuits left fixed by the different types of elements in S_3. For example, if the switching function $f(x_1, x_2, x_3)$ is fixed by the transposition (12) of the input variables, then $f(0,1,0) = f(1,1,0)$ and $f(0,1,1) = f(1,0,1)$. The values of f for the inputs $(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,1,0)$ and $(1,1,1)$ can be chosen arbitrary in 2^6 ways.

Table 2: The Action of S_3 on the Inputs of the Switches

Type of element $g \in S_3$	Number, S, of such elements	#fix g	S, #fix g
Identity	1	2^8	$2^8 = 256$
Transposition	3	2^6	$3.2^6 = 192$
3-cycle	2	2^4	$2.2^4 = 32$
	$\#S_3 = 6$		$\Sigma = 480$

By Burnside's theorem and Table 1.2, the number of non-equivalent circuits is $\frac{480}{\#}.S_3 = \frac{480}{6} = 80$, hence, there are 80 non-equivalent circuits that are obtained.

Conclusion

Permutation symmetry groups technique for counting switching circuits has been investigated using Polya-Burnside method of enumeration which shows the application of symmetry groups in electricity (physics). Since we are living in a three-dimensional world, these symmetry groups play a crucial role in the application of modern algebra to physics (switching circuits).

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