SEMANTICISM AS A METHODOLOGICAL INHIBITOR OF THE PRESENCE OF INTERPRETER SYSTEM IN LOGICIST EPISTEMOLOGY

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ABSTRACT

The study of logicism was carried out to ascertain the cause of the recorded failures of the logicists' logic. The method adopted for the study was phenomenological hermeneutics, which is a subset of content analysis. It was discovered that the central problem of logicism was the assumption of the validity of semanticism, with the attendant adoption of representational semantics as a substitute for epistemological demonstration. The consequence of this assumption is the inhibition of the presence of the interpreter system as a logically necessary component of the logicist epistemic demonstration. It was argued in line with Gödel's completeness thesis that an adequate epistemology must be behavioural and constructivist, which necessarily entails the inclusion of an interpreter system and is given symbolically as {U = (A, I)}.

Keywords: Logicism, Logic, Semanticism, Interpreter System, Behavioural Constructivism.

Introduction

The attempt to resolve the problem of failures of epistemological systems has not, until contemporary times, been addressed, other than by the presentation of privileged representations and cognitively superior bases for the justification of propositional knowledge. Quine (1969) moved the epistemological problem from simply presenting a privileged basis for propositions to questioning the theoretical assumption underpinning all proposals of bases. This assumption is the correctness of the absolute objective correspondence between language and the material universe, excluding all subject input interferences. It is this tendency—that knowledge is plastered on the senses directly from the material universe, with the subject as a passive receptacle—that secretly underlies epistemological research. This methodological orientation confuses epistemology with representational semantics and, in most cases, substitutes its programme for that of epistemology in the history of philosophy.

In taking up the problem of the epistemic axiom(s) that underlie traditional epistemology and extending its tentacles to the foundations of mathematics and logicism, the paper is following one of the current trends in epistemological history, a trend that questions discriminatory foundationalism. The new trend questions presumptive philosophizing, which is bereft of critical

analysis of all aspects of the inquiry to permit the data (text) of analysis to speak for itself. It is only in this way that truth-based epistemology can be discovered and the defence of armed-chair presumptuousness obviated. The trend could here be understood to be what some call *Phenomenological Hermeneutics*. It is a method of bracketing (suspending) all previous pieces of information, biases, and presumptions about the data (text) of the inquiry in the course of its interpretation, so as to allow the data (text) to speak for itself. Phenomenological hermeneutics is a subset of *Content Analysis*.

Statement of Thesis of the Essay

The paper discovers that the underlying and unstated philosophic assumption of the logicists is the validity of the picture theory of language, which influenced their conception of logic, nay, of set theory, and mathematics after subjecting the texts of logicism in the foundation of mathematics to phenomenological hermeneutic analysis. Logicism is therefore founded on logical atomism, which is again founded on faulty empiricism, which is a failed attempt to deviate from Platonism. Platonism is very much alive with us via *semanticism*; that is why logicism failed just like previous epistemological theories. Logical atomism and empiricism are forms of *Semanticism*. *Semanticism* is used here to refer to the assumption that since knowledge is couched in language, the referents of linguistic tokens are the same as the foundations of truth and its conclusive justification. Hence, the thesis of the essay is that the attendant platonism (*semanticism*) of mathematical entities in logicism, which led to the exclusion of subjective contribution or the absence of the interpreter system in logicist semantics, is responsible for the erroneous interpretation of numbers as empiricist elements in set theory. This essay defines the interpreter system as the sum of all subjective inputs that are not provided in any attempt to achieve a one-to-one correspondence matching of material experience with its corresponding linguistic output in semantic analysis. These subjective inputs are the subject's logical, non-abstract activities, the existence and structure of which can be gleaned when language is studied in speaking. The novelty of this essay, therefore, lies just in this thesis statement.

A Review of Select Diagnostic Analysis of The Logicist Conundrum

Williard Quine, in his *Ontological Relativity and Other Essays* (1969), opined that epistemology is concerned with the foundations of knowledge. Following Williard Orman Quine, G. O. Ozumba in his *Understanding the Philosophy of W. V. O. Quine* (2002) argues not only for the possibility of foundational epistemology but also for its uniqueness as the only legitimate epistemology. Foundation is the basis or ground according to which a belief is held to be true or justified in being held. For as Hetherington argues in *Knowledge Puzzles: An Introduction to Epistemology* (1996), knowledge entails truth. Philosophers have struggled with the question of what constitutes the domain of truth. According to theories of language exposed in Inyang Udofot's *English Semantics* (1998), truth is a property of language. This appears to be based on Donald Davidson's theory that identified truth with meaning in "Truth and Meaning" (1972). So, if meaning is understood in terms of "usage," as in Wittgenstein's *Philosophical Investigations* (1993), then truth is the satisfaction of the initial ontological permutation of a linguistic community. It implies a reduction of epistemology to semantics, which is, sadly, semanticism. This semanticism in epistemological analysis has been the major approach in history, such that no epistemologist questioned this basic epistemic methodological axiom governing the whole

traditionalist programme until Quine consistently questioned it. The point missed by semanticism is that epistemology is not semantics. Although semantics play an important role in epistemic analysis, epistemology cannot be reduced to semantics.

Yet traditional epistemology is replete with systems that are solely semanticist. A careful summary of the programme of epistemology in the History of Philosophy reveals the semantic approach. As a result, when Kant (1965) attempted to question and change the direction of epistemology, he saw the need to return to semanticism as a paradigm for demonstrating his epistemology. The case is not even better for Quine (1969), as the latter also demonstrated an epistemology laden with unresolved nominalism, an indication of a return to traditionalist semanticism. Thus, the Quinean proposal for the investigation of conceptualization was never achieved by Quine himself. The progress of that programme was marred by his nominalism. But that proposal transcended all previous attempts, which were characterised by seeking to render privileged representations with a certain yet unknown presupposition of a yet unknown epistemic axiom (semanticism), by asking questions concerning the truth of the axiom in general and the discovery of a true axiom in particular. But unfortunately, Quine never carried out that project successfully. So, the gap is still unfilled. It is that question that is the concern of this paper. The present review of the literature is to show not only what scholars have done but that no one has actually asked or answered that question since Quine for epistemology in general or logicism in particular.

What are the basic axioms, methodologies, and orientations governing traditional epistemological programme? What impact has it had on the resulting epistemological systems? Is it an adequate epistemological axiom? If it is, how do we explain the scepticism that colours traditional epistemology? By extension, does this orientation have an effect on the foundations of mathematics in general and logicism in particular? Thus, could it be used as explanation for the skepticisms in the logicist programme? These questions have not been answered yet. Even scholars in logic and the foundations of mathematics rarely address the matter from this standpoint.

One of the most prominent writers in logic and the foundations of mathematics is Stephan Korner. In his 1971 book, *Philosophy of Mathematics: An Introduction*, Stephan Korner situates the logicist programme within the epistemological structure of the synthetic-analytic bifurcation of knowledge. According to him, the recognition of the aprioricity of mathematics and logic is the basis for the logical reduction of the former to the latter. Korner argues, that the possession of one or more similar qualities is not an adequate basis for the establishment of the presence of identicals; otherwise, rational theology would be logic, because it is also an a priori science. In the analysis of the extension of mathematical concepts, Korner argues that logicism conflates pure mathematics with applied mathematics. According to him, the logicians have failed to make a distinction between *1 + 1 = 2* and '*one apple and one apple are two apples*." Hence, he tends to accuse the logicians of mathematical empiricism. Unfortunately, Korner has not asked for the epistemological basis of the metaphysics, which bifurcates knowledge into synthetic and analytic. Hence, he failed to question the basis for the logicist-empirical orientation, which is also responsible for the reduction of mathematics to logic and the nominalism of classes as presented by Russell.

Michael Resnik's problem with logicism in *Frege and the Philosophy of Mathematics* (1980) goes even deeper, as he refuses to recognise the adequacy of the logical-empirical orientation,

especially as long as it has failed to give an adequate reference to numbers. The argument derives from Resnik's belief that set theory is open to an infinite number of isomorphisms, arithmetic being one of them. Hence, to argue that set theory captures arithmetic is not different from saying that talk about God could be captured in a formalised ontology. However, God cannot be understood simply by speaking about him in another language. In the same vein, Resnik argues that what numbers in arithmetic are is unknown. Even when they are called classes, it is difficult to say what they are. According to Resnik, the complications get even deeper in Russell's nominalism, which advocates a class and no class theory. Thus, if numbers are classes and classes are unknown, then what numbers are is also unknown. In that sense, the foundations of both mathematics and logic are unknown. Resnik had to come to a halt at this point and ask why the sudden epistemological stalemate. Resnik, however, failed to identify the problems found in logicist empiricism with the invalidity of semanticism as an epistemic axiom.

In the instance of such an epistemological stalemate, the only options open to the epistemologist are either platonism, conceptualism, or nominalism. Russell took to nominalism. Hence, like language, logic and mathematics are understood as conventional. After all, in that system, logic is a meta-language. Classes are shown in his 1998 *Introduction to Mathematical Philosophy* to be fictitious logical objects. Gila Sher is enthralled by such conventionalism as an alternative to outright Platonism. In *The Bounds of Logic: A Generalized Viewpoint* (1991), she argues for logical conventionalism, which eventually will promote ontological arbitrariness. Hence, logicism, a reputed programme whose theoretical foundation was to reduce mathematics to logic, is theoretically trivial. Sher argues that any argument that is properly Kantian with respect to logic is not properly Carnapian. According to Kantian logic, Aristotle had completed the task of outlining the principles governing human reason. But Carnap, as Sher argues, allows for the proliferation of logic. The task for Carnap is not to decode which of the different systems is the right logic but to examine their formal properties and the possibilities for their interpretation and application in science (Sher, 1991). Sher's thesis for logic is one that understands the science as the study of formal systems. It is concerned with the empirical object, which is a particular formal system. This is different from the ideas Frege and Russell had about logic. Their logic was fundamental, not arbitrary formalism empiricism.

It was on the basis of this that Russell, in his *Principles of Mathematics* (1992), rejected Peano's idea of a formal system for logic. Peano had argued that the axioms of the system need not be the most basic. What was needed was a demonstration that all the theorems of the system were derivable from the axioms. Russell argued, however, that the axioms of logic must be the most basic, beyond which there are no others. In short, they must be those that are invariant in all models. In analysing that system, Tarski argued in "What are Logical Notions?" (1986) that logical notions are those that are invariant in a type structure (formal systems) over a given domain of individuals, irrespective of the arbitrary permutations of the domain. This agrees with Russell's notion of a logical notion and disagrees with Sher's notion. So, by using her notion of logic to criticise the logicist programme, Sher's criticisms become irrelevant.

Nevertheless, Solomon Feferman, in "Logic, Logics, and Logicism" (2005), argues that Russell's logic in the *Principia Mathematica* (1978) is not invariant in its universal application. According to him, the only logic that enjoys the privilege of invariance is the first-order predicate calculus. Consequently, from the point of view of universal invariance, which Feferman designates

"homomorphism," mathematics does not reduce to logic because first-order calculus cannot carry the weight of mathematics. Feferman, however, does not worry about the epistemological orientation that allows a mere formalised ontology to be called a logic. He proposes that the study of logic should be more concerned with rational processes.

Volker Peckhaus and Edward Zalta are two scholars who accept the ontological orientation that underpins the formalism of logicism. Peckhaus, in "Calculus Ratiocinator vs. Characteristica Universalis" (2005), believes that even as a universal characteristic, the logician's system also contains a calculus ratiocinator. Hence, it is a complete universal system. Peckhaus has, however, failed to notice that the universal characteristics of the Fregean triumph over Boole's calculus are properly ontological in their reference and not logical. This implication is emphasised by Erich Reck in "Frege's Natural Numbers: Motivations and Modifications" (2005). According to Reck, the motivations for Frege's system are the old pluralist orientations, which sought to reduce mathematics to geometry. Reck identifies the works of Aristotle, Euclid, Mill, and Weierstrass in this tradition. He also mentioned that the plurality conception was revived in the nineteenth century's foundations of analysis. Hence, it was this idea that led to the understanding of numbers in terms of the extensions of class. Thus, the moment the antinomies were discovered, Frege opened up the logicalist pretensions and asked Russell whether it would not be better to base arithmetic on geometry. The limitation in Reck's analysis is his restriction of the investigations to the ontological motivations for the logicist thesis without asking questions concerning the presupposed metaphysics of the epistemic research project, which defines the epistemic axioms guiding demonstrations in the programme as semanticist. Besides, logicism purports itself to be faced with set theoretic contradictions. But scholars have not asked questions concerning the relationship of the contradictions to the logicist's presupposed epistemic axiom and its methodology.

The ontological pretensions of logicism have been laid bare in Edward Zalta's "Neo-logicism: An ontological Reduction of Mathematics to Metaphysics" (2005). According to Zalta, the logicist program, according to which mathematics reduces to logic, has failed. He argues that mathematics reduces to logic if logic has objects of some sort. Zalta argues that classes are not logical objects. He contends that logic has no objects after all. According to him, the reduction of mathematics to a logic of class is a reduction to metaphysics, the referent of which is Platonic. As a result, Zalta's interpretation of logicism is that it is a platonic metaphysics. Numbers, Zalta argues, are platonic objects. He opines that such objectivity overcomes the meaninglessness of the formalism of the Russellian sort and does not permit the Gödelian consequence for mathematics. Nevertheless, Zalta's basis for rejecting the logicist thesis seems genuine. But his own programme equally stands trial in the present research because of its entanglement with semanticism.

The existence problem in logicist logic, which has led to the interpretation of the system as a metaphysics, was also observed by Jose Ferreiros in "The Crisis in the Foundations of Mathematics" (2005). According to Ferreiros, the axiomatic method's existence proof and the corresponding assumption of ideal existence, in order to satisfy the Dirichlet proposal and put thought into calculation, when combined, gave impetus to logicism. The impetus derives from a prior acceptance of set theory as logic. The acceptance of the ideal existence of concepts as the domain of mathematics meant the acceptance of mathematics as identical with set theory. But the problem that arises here is whether numbers are platonic forms, classes, or sets? Or whether these forms are required for logic's legitimacy? Ferreiros argues, that the problem with the

resultant logicism is that its ontological content deprives it of its pure logical character. Hence, he observes that Ramsey's attempt to prove the logical character of the existence axioms of infinity, choice, and reducibility was inconclusive, and Russell's attempt to rescue logicism from the paradoxes remains unconvincing. Ferreiros does not seek to answer such deep epistemological questions as the ones proposed here.

Arguments akin to the metaphysical accusations of logicism are found in the work of Raymond Wilder, entitled *Introduction to the Foundations of Mathematics* (1955). Wilder refused to address the problem, whether mathematics reduces to logic or not. He contends that the logicist program's failure is rooted in its use of the axioms of infinity, reducibility, and choice, which are actually about extensions rather than pure logic, to achieve technical mathematical goals. Wilder's analysis identifies ontological elements in the *Principia Mathematica* (1978), but he fails to link the false ontology with semanticism. Even when scholars came close to identifying some logic failures, no writer studied thus far was able to identify the absence of the interpreter system, let alone relate it to the assumption of semanticism as a valid methodological axiom as a key problem in logicist epistemology.

The Formal Structure of the Interpreter System in Complete Epistemology of Language Systems

It would amount to dishonesty to claim that logicians are unaware of the difficulties associated with their system. However, because they are unable to identify such with their methodological assumptions, their approach has been to use a fragmentary emergency life-support model to manage the system's fallout. One such approach is that proposed by Gregory Landini. Landini, in "Logic in Russell's Principles of Mathematics" (1996), argues that as long as the logic of *The Principles of Mathematics* (1992) is semantically complete, it is also axiomatically complete. Landini's notion of the semantic completeness of the logical system in question is founded on its axiomatic categoricity relative to sentential calculus. Even though categoricity has come to stay as the basic test for relative axiomatic and semantic completeness, it is even known and accepted in the community of mathematical logicians that total completeness, which is the goal of all axiomatic systems, is achievable with the presupposition of not only the semantic model of the system but also with reference to the domain, the principles of comprehension, and the way of comprehension itself. The principles and manner of comprehensiveness presuppose the interpreter function. So, even if Landini succeeds in his argument about relative completeness, the system remains completely incomplete.

What Russell and Landini seek to achieve in logicism is achievable if Thomas Jech's presuppositions are taken seriously. In his *Set Theory* (1987), Thomas Jech has shown that the problem of incompleteness could be overcome if the system is presented such that in a language *L*, the knowledge system *U = (A, I)*, where *A* is the domain and *I*, the interpreter function. Semantics, on the other hand, presupposes the establishment of A given U, thereby limiting epistemology to representational semantics. Gödel exposed the weakness of such an approach and proved the necessity of the interpreter system as a necessary external component of the linguistic expression of knowledge. It was *I* that Gödel proved to be necessary in comprehensive epistemology, though it can never appear in a formal system. If it does, it cannot be proven in it. But unfortunately, traditional epistemologies persist in the demonstration of the foundations of knowledge by presenting only the relationship between *U* and *A*. Logicism is also known as epistemology, and this is the bane of its project.

Conclusion

Semanticism is itself limited with respect to the objective of the epistemic research project. Given that epistemology is concerned with establishing the foundation of knowledge, which involves the provision of a comprehensive and conclusive disclosure of the relationship between the meagre input of our knowledge claim and its outrageous output found in language, the method of representational semantics would be inadequate for an epistemology. An adequate epistemology would be of the behavioural and constructivist type, which takes into account how the human subject of the study weaves his knowledge (*U*) from experience (*A)* with the aid of its logical and mathematical apparatus (*I)*, believing that we share the same position in the universe as it does.

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